A New Stress Analysis Method for Hypoid Gear Drives

Jui S. Chen
American Axle & Manufacturing, Inc
2965 Technology Dr
Rochester Hills, MI 48309, USA

A hypoid gear drive transforms torque between crossed axes and is widely used in front and rear axles. Current requirements for greater engine power and higher fuel efficiency mean hypoid gear axles must have a higher power density. Therefore, engineers need a tool, which efficiently and accurately optimizes hypoid gear stress, to check design feasibility before prototyping and testing is done. This paper proposes a new, practical methodology that more accurately and efficiently calculates the stress of hypoid gears under loading.

Keywords: Hypoid Gear, Stress Analysis

INTRODUCTION

Historically, the classical calculation of gear-tooth bending stress has centered on the use of cantilever beam theory to simulate stress conditions encountered in gear teeth. Most of the research in this area has involved spur and helical gears. Kudrjavtsev (1), Winter (2) and Niemann (3) are among the very few to develop analytical formulas for calculating hypoid gear bending stress by modifying the formula for spur and helical gears. Additionally, Gleason Works (4) developed the Q Factor method for calculating the bending stress of hypoid gear drives. Each of these methods use simplified modeling and compensates by experiments. Although useful, the results are not extensive enough to assist engineers in improving the design.

Thanks to the increasing power of computers, finite element analysis (FEA) can now predict stress much more accurately. For hypoid gears with lengthwise curvature and asymmetric shapes, only a full 3-D FEA can properly predict load stress. Wilcox in 1981 (5) and 1997 (6); Chen in 1993 (7), and Handschuh and Bibel in 1999 (8) provided a 3-D FEA methodology to calculate hypoid gear stress. Wilcox used H-adaptive finite element theory and Weber’s tooth contact model that is outlined by Yau (9). Handschuh used non-penetration constraints between the pinion and gear surfaces. Chen determined the contact forces, contact areas and calculated the bending stresses on the pinion and gear separately.

Since there is not an easy and accurate tool available, the strength of a hypoid gear drive is usually validated by testing a full axle in a dynamometer. The timing from the development of a new hypoid gear set through its complete testing usually takes from one to six months. The test result includes not only the design parameters, but also the process variation. Based on the timing and cost restrictions, engineers seldom have a chance to optimize the design.

The purpose of this paper is to provide a simple method that enables engineers to evaluate the strength of a hypoid gear design. The flow chart for this method is as shown in Figure 1. The total time to complete one analysis by this method is approximately eight hours. The process time is reasonable and can be reduced by further study.

GENERATION OF HYPOID GEAR SOLID MODELING

Gleason’s face-mill fixed-setting generation method is considered in this paper. The points on the gear and pinion surfaces can be described mathematically by using the kinematics of the manufacturing process. A surface generation program (SURFACE) is developed to generate the gear and pinion tooth surfaces based on the theory of gearing (10). The surfaces can then be used to create the slot solid between teeth. A blank generation program (BLANK) can create the input data for a CAD program like Unigraphics to generate the gear and pinion blanks without teeth. Teeth can be created by cutting the blank with the slot solid.

GENERATION OF HYPOID PINION TOOTH SURFACE

The pinion surface is determined by the generation motion between the machine cradle and the pinion, as well as the geometry of the cutter blades. The cutter geometry is a cone surface as shown in Figure 2. The equation for the cone surface is represented by:

\[
\begin{align*}
    r_{f1}(S_F, \theta_F) = & \left[ \begin{array}{c}
    (R_{cp} + S_F \sin \alpha_F) \cos \theta_F \\
    (R_{cp} + S_F \sin \alpha_F) \sin \theta_F \\
    -S_F \cos \alpha_F
    \end{array} \right] 
\end{align*}
\]

(1)

Here, \( R_{cp} \) is the cutter point radius. \( S_F \) and \( \theta_F \) locate the cone surface points. \( \alpha \) is the blade angle. \( r_{f1} \) is the position vector represented in coordinate system \( f1 \).
The unit normal to the cone surface is represented by:

\[ \mathbf{n}_{f1} = \frac{\frac{\partial r_{f1}}{\partial S_F} \times \frac{\partial r_{f1}}{\partial \theta_F}}{\left| \frac{\partial r_{f1}}{\partial S_F} \times \frac{\partial r_{f1}}{\partial \theta_F} \right|} \quad (2) \]

The pinion tooth surface is determined as the envelope to the family of the cutter surfaces. The family of the cutter surfaces is generated by coordinate transformation and is represented as:

\[ r_i(S_F, \theta_F, \phi_F) = M_{ij} r_m M_{mi} M_{mj} M_{mi}(S_F, \theta_F) \quad (3) \]

where \( M_{ij} \) is the transformation matrix that describes the coordinate transformation from coordinate \( j \) to coordinate \( i \) (7). \( \phi_F \) is the generation parameter.

The necessary condition of the existence of the envelope to the family of surfaces is the **equation of meshing** (10):

\[ f(S_F, \theta_F, \phi_F) = (\frac{\partial r_i}{\partial S_F} \times \frac{\partial r_i}{\partial \theta_F}) \cdot \frac{\partial r_i}{\partial \phi_F} = 0 \quad (4) \]

Considering Equations (3) and (4) together, the pinion tooth surface is determined and can be represented as a function vector of two parameters \( r_i(\theta_F, \phi_F) \). Since concave and convex surfaces are cut separately, the relative position of the pinion tooth is determined by the backlash between the gear and pinion.

**GENERATION OF HYPOID GEAR TOOTH SURFACE**

The gear cutting cone surface is the same as the pinion cutting cone surface, but the generating process is different. The gear is generated by formate method, so the gear tooth surface is the same as the cutter cone surface.

The gear tooth surface and its normal can be determined by:
where \( \mathbf{r}_2 \) is the position vector of the cutting cone surface.

**GENERATION OF HYPOID PINION AND GEAR BLANK**

An Excel program BLANK is developed to generate the blank data for a CAD program. The basic gear dimension data can be “cut” and “paste” from the Gleason program to the BLANK program.

**GENERATION OF SOLID MODELING**

The CAD program can create the gear and pinion solid modeling as shown in Figure 3 by reading surface data files from the SURFACE program.

**TOOTH CONTACT ANALYSIS FOR HYPOID GEAR DRIVE**

A tooth contact analysis (TCA) program is developed to provide the initial contact position for FEA. Before performing FEA, the pairs of contact surfaces should be put in separate positions. The simulation of meshing of the pinion and gear tooth surfaces is based on the condition of surface continuous tangency. The tangency of gear and pinion surfaces considered in the fixed coordinate system \( p \) is represented by the following equations

\[
\mathbf{r}_p(S_G, \theta_G, \phi_G) = M_{2m_p}M_{m_p}M_{r_p} \mathbf{r}_p(S_G, \theta_G) \quad (5)
\]

\[
\mathbf{n}_2 = \frac{\mathbf{F}_2 \times \mathbf{F}_1}{\sqrt{|\mathbf{F}_2 \times \mathbf{F}_1|^2}} \quad (6)
\]

The superscripts “(1)” and “(2)” designate the pinion and the gear, respectively. \( \phi_1 \) and \( \phi_2 \) are the angles of rotation of the pinion and gear.

The system of equations (7) and (8) can be solved by the function of \( \theta_F(\phi_2'), \phi_F(\phi_2'), \phi_1(\phi_2'), S_G(\phi_2'), \) and \( \theta_G(\phi_2'). \)

**FINITE ELEMENT MODELING**

Two different solid elements are investigated in this paper. The first element is an eight-node hexahedral element as shown in Figure 4. The second element is a ten-node tetrahedron element as shown in Figure 5. ABAQUS (11) recommends the eight-node hexahedral element with incompatible mode as the best element choice for contact with bending problem. The failure mode of hypoid gears is bending fatigue in the root of the tooth, so the eight-node hexahedral element with incompatible mode is used in this study. The use of hexahedral elements allows more control and accuracy. On the other hand, the modified ten-node tetrahedron element is also considered in this study since the geometry of the tooth is complicated and difficult to use with the hexahedral element.
The pinion shaft is modeled by a 2-node beam element to simplify the analysis. The input pinion torque in this analysis is 12,341 in-lb. and is applied to the tail end of shaft. The thrust bearings of the pinion are simulated by constraining translation on the position of the pinion journals. The bottom of the ring gear is fully constrained by 10 bolts. Three pairs of surfaces in contact are assumed in the hexahedral element modeling, however, the result shows only two of them have contact condition. Two pairs of surfaces in contact are assumed in the tetrahedral element modeling because of the lesson learned in the previous analysis.

Standard steel modulus of elasticity 30,000 ksi and Possion’s ratio 0.292 are used in the analysis. Standard ABAQUS code is used to perform the stress analysis. Static nonlinear contact analysis with small sliding contact options is specified. Two steps are used to simulate the contact process. In the first step, the pinion rotates at a small angle to have initial contact with the gear. In the second step, the input torque is applied to the pinion shaft to complete the analysis.

RESULTS AND DISCUSSION

The result of two analyses is tabulated in Table 1. Analysis one using a hexahedral eight-node element with incompatible mode is the best choice for the contact bending problem. Analysis two using a modified tetrahedron element reduces the time to build the modeling from 40 hours to 3 hours. The maximum principal stress contours of gears are shown in Figures 6 and 7. The result shows the maximum bending stresses for two gears are very close (less than 1%). The maximum principal stresses of the pinion are shown in Figures 8 and 9. The result shows the maximum bending of pinions has a greater difference (more than 37%).

The reason for this larger discrepancy could be that finer meshes are needed for both modeling because there is more curvature in the pinion tooth than in the gear tooth. The preliminary results show some effort is still needed to reduce the difference between two different elements, but it also shows tetrahedron elements created by auto-mesh is a promising method to solve the hypoid gear stress problem. The difference between maximum von Mises stresses as determined by the two methods is much larger than maximum principal stresses. Since the failure mode of hypoid gear is bending fatigue in the root, maximum von Mises stress is not the concern in this paper.

The successful implementation of this methodology requires five key steps be taken and properly controlled.

1. Generate hypoid 3-D solid modeling efficiently.
2. Determine the initial contact position before loading.
3. Create finite element meshes efficiently.
4. Determine stress under loading by proper nonlinear contact solver.
5. Develop an after-heat-treat hypoid gear product as close as possible to the theoretical one. This allows results to be validated by testing real parts.

The next critical effort is to perform experiments and compare the results with FEA results. Both elements discussed in this paper have good potential to perform accurate and efficient analysis.

<table>
<thead>
<tr>
<th>Element Type</th>
<th>Analysis 1</th>
<th>Analysis 2</th>
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<tbody>
<tr>
<td>Number of Element</td>
<td>29652</td>
<td>16605</td>
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<tr>
<td>Number of Node</td>
<td>34810</td>
<td>27557</td>
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<tr>
<td>Time to Build Modeling (Hours)</td>
<td>40</td>
<td>3</td>
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<tr>
<td>Time to Run Analysis (Hours)</td>
<td>3.7</td>
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<td>Maximum Bending Stress - Gear (ksi)</td>
<td>135</td>
<td>134</td>
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<tr>
<td>Maximum Bending Stress - Pinion (ksi)</td>
<td>86</td>
<td>118</td>
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<tr>
<td>Max Von Mises Stress - Gear (ksi)</td>
<td>255</td>
<td>266</td>
</tr>
<tr>
<td>Max Von Mises Stress - Pinion (ksi)</td>
<td>83</td>
<td>223</td>
</tr>
</tbody>
</table>

Table 1 - Result Comparison between Two Analysis
CONCLUSION

A methodology is proposed for industrial engineers to solve stress analysis for hypoid gear drives. The preliminary result shows the method is promising. Additional experiments are needed to verify the analysis result, and more effort is needed to improve the accuracy and efficiency of FEA modeling.

REFERENCES


